Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

with stately dignity and aristocratical composure; his exhortation to the prizemen to persevere in their studies with unabated ardour, was eloquent and impressive; and he concluded his address by paying a compliment to Mr. Brougham, not less beautiful than it was deserved.

W.

⁂ Our valued correspondent seems in the foregoing article to differ from us in *some* of his political views—while conscientiously holding our own, we cannot have any objection to put forward his—for which we desire not to be held responsible.—Ed.

LINES WRITTEN AT SEA.

I love to see the milk-white foam,
I love to see it rise,
As it sparkles in the sunny beam,
As it dances towards the skies.

I love the fair receding shore,
With groves and villas gay,
I love to see the water birds
Dip lightly in the spray.

I love the green and glassy waves,
I love the changing clouds—
The glories of the sun-lit noon,
And ev'ning's sober shrouds.

To nature, thro' her wide domains,
My ceaseless vows I bring;
Whether enthron'd in winter's storms,
Or blooming in the spring.

Yes, nature—yes, I bow to thee
Alike, in sun and snow—
In ev'ry varying mood and form,
My spirit owns thy glow.

Anna Maria.

TRIGONOMETRY—DIFFERENTIAL & INTEGRAL CALCULUS.*

We are old enough to remember when a treatise on the Differential and Integral Calculus would have been received by the students of the University of Dublin, much as the monks of yore are said to have received books in the Greek language—with the self-consoling remark, *Græcum est legi non potest.* The change that has since been wrought, is such as to render almost incredible how short the period is since the doctrine of quadratic equations was looked upon as the very summit of an undergraduate's analytical attainments, and a portion of Maclaurin's fluxions the greatest extent to which even fellowship candidates were wont to penetrate into the region of transcendental science. If Dublin was behind her British sisters in the introduction of what were then called "the French Mathematics," the rapidity of her progress—unequalled, we venture to assert, by that of Cambridge, and far surpassing that of Oxford—has amply proved that inferiority in talent was not the real cause of her delinquency. In confirmation of this opinion we shall merely state these facts:—Euclid's Elements were first made a part of the undergraduate course, we believe, in the time of Provost Hutcheson; and this, together with a very small portion of Algebra, which was not introduced till long after, was all the mathematics read by undergraduates till about the year 1812. In 1816, the gold medal course embraced Trigonometry, plane and spherical; analytic geometry, including the conic sections; the differential calculus; the theory of central forces, treated analytically; together with the corresponding parts of the first book of Newton's principia.* At present, the candidates for the same honour read the above subjects in a way much more extensive, together with a large portion of the integral calculus, Poisson's treatise on mechanics, Dr. Lloyd's excellent work on the same subject, the mathematical theory of geodetical operations, and some additional portions of Newton's principia. This has been very nearly the state of the course for about four years; while the introduction of Brinkley's astronomy and Dr. Lloyd's mechanics into the general College course is a sufficient proof that the improvement we speak of has not been entirely confined to candidates for the higher honours. Still, however, it must be admitted that large room for further improvement continues to exist: the mathematical department does not harmonize with the physical part of the course; and the interval between the science read for the medal and that necessary for undergraduate honours, is, we think, much greater than it should be. How far these defects may account for the *silence* that is so often (and not altogether without reason) objected to our University, is a question that would well deserve to be deeply considered. The fact, however, is undeniable, that Dublin *does not* produce the men that distinguish the Universities of England and Scotland—the fault we know does not lie in any inferiority of Irish talent—we are willing to admit that it may in part arise from the circumstance that Dublin is in Ireland, and therefore its University unfashionable with the nobility, and unpatronised by government: but the true cause, we are persuaded, lies in the serious defects of the system to which the fellows of Trinity College are tied down. To point out those defects, and their remedies, is a work that *for the present* at least, we must decline. In fact, we feel strongly our inadequacy to the task; and, contrary to the usual tactics of reviewers, who seldom hesitate to pronounce magisterially *de omni scibili*, we honestly avow our in capacity. But lest what we have said should suffer with our readers from this piece of candour, we shall quote the complaint of a distinguished member of the College, whose competency to give an opinion on this important subject none will dispute—Dr. ROMNEY ROBINSON thus speaks in the preface to his Mechanics:—

---

* Elements of Plain and Spherical Trigonometry; with the first principles of Analytic Geometry. By James Thomson, LL.D. Professor of Mathematics in Belfast College. Second edition. Belfast, 1830.

An Introduction to the Differential and Integral Calculus; with an Appendix, illustrative of the Theory of Curves. By James Thomson, LL.D. Professor of Mathematics in Belfast College. Belfast: Simms and M'Intyre: 1831.

> "Under the system pursued at present in Trinity College, its fellows can scarcely be expected to devote themselves to any work of research, or even of compilation; constantly employed in the duties of tuition, which harass the mind more than the most abstract studies, they can have but little inclination, at the close of the day, to commence a new career of labour. How different is this from the state of the English Universities, where the tutors constitute a very small part of the body, and the remainder have both leisure and incitement to pursue their peculiar studies, and increase the literary fame of their Alma Mater by their publications. In the present case the author happened to be less occupied than most of his brethren, yet he was engaged from seven to eight hours daily in academical duties for the year during which he composed this work."

But we must not forget our immediate subject. The titles of the works at the head of this article are enough to prove that the progress of science in this country has not been confined to the University alone. The lectures of Dr. Thomson have done much to propagate a very enlarged acquaintance with analysis among the students of the Belfast Institution: and we regard this circumstance with the more pleasure, as that establishment sends, we believe, but a small proportion of its *elèves* to the University; and consequently the scientific knowledge it disseminates is among a class entirely distinct from those who are initiated into the same mysteries in Trinity College.

---

* See the Dublin Problems lately published.

The treatise on Trigonometry, which stands first on our list, is an enlarged edition of a work originally drawn up by Dr. Thomson, with a less extensive object in view. Its execution does the author great credit, and we cannot but add, that the typography of both his works (executed in Belfast) puts to shame some specimens of mathematical printing that we have lately seen from the press of the University. Our limits prevent our entering so fully upon the treatise before us, as its merits deserve; and, as it presents no peculiarity, we prefer expending what space we have left in recommending the other work to the notice of our mathematical friends.

In the treatise on the Calculus, Dr. Thomson has (so far as we know) given the first example of an English elementary work on that subject, founded entirely on the principles of Lagrange. No one can read the *Calcul des Fonctions* of that great mathematician, without being attracted by the singular beauty and symmetry of the method he has there developed. Whoever loves to worship the spirit of the modern analysis, may there approach her very essence. And yet we are not sure that Lagrange's principle is the best adapted to the purposes of elementary instruction; not that we conceive it impracticable to reduce the first steps of that principle to the degree of simplicity necessary for the initiation of a tyro, (did we object to it on this ground, Dr. Thomson's success would refute our opinion;) but we do decidedly think, that the *first notions* derived from this method, are not the best calculated to give correct ideas of the true metaphysical character of the differential calculus. Lagrange, in the view we take of his theory, has disguised that character, and substituted another for it—the characteristic we allude to is the principle of *continuity*, of which the germ is to be found in the ancient method of exhaustions: and we are decidedly of opinion, that Newton's theory of fluxions has represented this better than any other system. As to the objection that notions foreign to pure mathematics, are necessary to the fluxional method, we count it of no real weight. In fact, Newton has made use of the terms, *flowing of a point*, and *velocity*, to express that very principle, which, as we have said, constitutes the true character of the differential calculus. If the notion of time be a convenient one for representing this principle to our conceptions, we cannot see what there is in it inconsistent with pure mathematics. Were it not that we are in the habit of considering time and motion in connection with physics, we never would conceive any such inconsistency to exist. Newton makes use of time as a kind of *independent variable*, to which he refers his differentials or fluxions. From the moment we arrive at the fluxional co-efficient, the idea of time is, as it were, eliminated; and the method of fluxions absolutely coincides with the method of prime and ultimate ratios, or limits, (which, by the way, is also Newton's, though it is often erroneously ascribed to D'Alembert,*) whereas Lagrange's method does not meet the other systems until we have demonstrated Taylor's theorem. The essential spirit of Lagrange's theory is to establish this theorem by pure analysis, and from it to deduce the first notions of what is, properly speaking, the differential calculus. This circumstance, we think, Dr. Thomson has overlooked: he deduces the notions of differentials, and differential co-efficients, *by arbitrary definition,* from the expansion of functions, and afterwards employs the rules of differentiation to establish the theorem of Taylor. Now Lagrange's idea was to deduce *the notion* of differentials, and differential co-efficients, from this theorem—and in so doing, he adhered rigidly to the spirit of analysis. he changed (as we said before) the principle of continuity into another principle more consonant to his great design—we mean the principle of *indetermination.* With Newton differentials were finite quantities, subject to a *flowing* or *continuous* variation—the *velocities,* in fact, of the variables: with Lagrange they were also *finite* quantities, but *essentially indeterminate.* Thus, then, these two methods arrive at the same result, and differ only in having arrived at it by different roads.

---

* Dr. Thomson has followed his predecessors in this error. "The method of limits, (he says,) *due to D'Alembert*, is free from some of the objections to which the methods of Newton and Leibnitz are liable."—p. 241. The fact is, that D'Alembert's

On the whole, then, we hazard the opinion, that the elementary principles of the calculus will never be presented under a perfectly unobjectionable form, until all the methods be combined—until it be made plain to the student, that the notion of time, of infinity, or of indetermination, are only auxiliaries, adopted as *mathematical representations* of the metaphysical principle of *continuous* variation. We can conceive the definitions of the science so modified as to make it plain to the student that such is the case; and we cannot but think, that the truest notion of the real metaphysical character of this wonderful offspring of genius, will be best had by employing, not now this principle, and again that, but by presenting *together* the three principles, in such a manner as to make it appear that they are but *aids* to our conceptions, and means whereby those conceptions can be *translated* into the language of alge-

---

method differs no more from Newton's prime and ultimate ratios, than Dr. Thomson's own method may be said to differ from that of Lagrange. D'Alembert has merely expressed Newton's doctrine in the Leibnitzian notation; and in his celebrated article in the Encyclopedie, lays no claim to any higher originality than this leaves him in possession of. Nor has he freed his method from the notion of *time*, which is considered so great an objection to Newton's, as will appear from the definition he gives of the differential co-efficient, viz.—"The ratio of the *simultaneous* increments of the function and variable" The true metaphysical difficulty in both the method of limits and that of prime and ultimate ratios is, that a limit is in fact a ratio whose terms have ceased to exist; and to this it is that Berkeley alludes, when he calls differentials of the higher orders, "the ghosts of departed quantities." This difficulty (if it be one) is avoided in the method of fluxions; but is increased, if any thing, in the differential method of Leibnitz. Newton, by means of time, and Lagrange, by the principle of *indetermination*, have represented differentials by *finite* quantities: and when we free the idea of velocity from all physical associations, and have reduced it to its pure mathematical conception, it absolutely coincides with the notion of indeterminate variation, employed by Lagrange to represent the law of continuity. In the method of Leibnitz, which is properly the *differential* method, he begins with the principle employed by the ancient geometers, viz.—variation by *finite* differences; and from this he derives the condition of *continuous* variation, by supposing the differences to become *infinitely small.* And here again, when we free our minds of the popular ideas attached to the word *infinite*, and come to examine the real *mathematical* notion which Leibnitz has employed that word to represent—"*on retombe,*" as the French significantly say—upon the very same idea that Newton, without any metaphysical obscurity, has exhibited by the analogy of time and velocity. In the method of limits, the notion of differentials does not occur at all, and differentiation is performed by differential co-efficients. The introduction of the Leibnitzian notion of differentials into the method of limits, has occasioned a difficulty being charged upon this method, from which its rigid principles (as presented by Newton) are entirely free.

bra. And we repeat our original assertion, that of the three we think the method of Lagrange, (admirable as it truly is, when considered as harmonizing with the spirit of pure analysis,) is the least adapted to give the tyro a correct notion of the metaphysical principle to which we have so often alluded.*

Although, therefore, we do not believe that the differential calculus has gained much, either in uniformity or in facility, by the plan followed by Dr. Thomson; yet we cannot but do him the justice to say, that he has executed it with great skill, and with much greater success than we would have expected, had we been informed of his design before we saw his book. He has completely succeeded in bringing the principle of Lagrange's method within the reach of any student tolerably familiar with common algebra; and in saying this, we give him no small praise. He derives the rules of differentiation, by easy development, from the first terms of the expanded differences; and then, by indeterminate co-efficients, combined with differentiation, deduces from the *form* of the series previously established, the successive co-efficients of Taylor's theorem. We have already endeavoured to state our objections to this manner of establishing that important formula; yet, in justice to Dr. Thomson, we cannot but admit, that these objections apply rather to the metaphysical rigour of the science, than to the purpose he appears to have had in view, which was not to furnish a complete treatise, but to prepare the student for larger works. And in this respect it matters little by what metaphysical principle the student is conducted to the end proposed, provided he meets with nothing to lead him from the truth. That our author has attained this object, we are fully satisfied; yet we cannot help retaining our opinion, that the differential method of Newton and Leibnitz is that which holds best to the memory, and best assists the student in applying the science to geometrical and physical investigations.

Dr. Thomson's treatise takes a more extended range than we had expected from its size before perusing it. He has succeeded in compressing into a small compass the elementary principles, not only of the differential and integral calculus, but also of the calculus of finite differences, direct and inverse, and the calculus of variations. The two last sections contain exercises in astronomy and geometry; and the appendix is occupied with the properties of conic sections, deduced from their equations, together with those of some transcendental curves, and concludes with an article on the methods by which other authors have established the science. On a passage in this article we have already animadverted, and we are glad that our limits prevent us saying more, as this is the only part of the work with which we are at all inclined to quarrel. We must not, however, enter into the subject, having already deluged our unscientific readers (if any such we have) with more mathematics than we at first intended, and having, we hope, said enough to convince all others that Dr. Thomson's works are eminently deserving of their attention.

---

* We quote the following passage from EULER, as in substance agreeing with the above observations, and we are glad to take shelter under the authority of a writer, whose qualifications for giving an opinion are beyond dispute, and who cannot be accused of national partialities :—"Quas nos quantitates variabiles vocamus, eas Angli nomine *magis idoneo* quantitates fluentes vocant." He adds, after alluding to the radical identity of the two systems of Newton and Leibnitz—"Equidem Anglos in formulis loquendi lubenter imitarer, sed signa quibus nos utimur illorum signis longe anteferenda videntur." *Euleri Calc. Integ.* vol. i, p. 3.